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Stability of parameters characterizing the rates of return on shares

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Abstract:

Aim: The aim of the paper is to measure risk and profitability of selected financial instruments.

Design/Research method: the theory of order statistics, with a particular emphasis on extreme statistics, was employed in the research. The suitability of Wiener process and Ito's process for the description of stock price volatility was investigated.

Conclusions/Findings:. Based on KGHM shares, the stability of parameters of the rates of return distribution was found.

Originality/Value of the article: Two separate theories within the scope of stochastic processes, extreme statistics theories and Ito's processes, were combined in the paper. The application of the appropriate distribution of the rate of change is a new and original proposal.

Keywords: rate of return, Wiener process, Black-Scholes model, stochastic differential, extreme statistics.
JEL: C12, C58

1. Introduction

The aim of the paper is to measure risk and profitability of selected financial instruments. In the analysis of financial instruments (especially stocks), the rates of return are of particular importance. This refers to the portfolio analysis, where, next to the risk measure, the rate of return is the basic parameter characterizing stocks. The value of this parameter is crucial in specifying the portfolio parameters. In Markovitz classical theory (Haugen 1996), there is a very constraining assumption on the stability of return and risk measures. The instability of parameters

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can nullify the benefits arising from the portfolio analysis.

Another important issue involved in the application of parameters characterizing income and risk is pricing of securities. In this paper, a particular emphasis will be put on the assumptions underlying the Black-Scholes model for option pricing (Weron, Weron 1999). These assumptions allow for constructing a test based on order statistics (Czekała 1998).

The so called technical analysis is a theory extremely popular among investors (not only the individual ones). This is due to the fact that the signals coming from it are simple and easy to apply by investors on the stock exchange. However, the effectiveness of the analysis is disputable, to say the least, and investors who use it often tie its findings with the analysis of market psychology. An analysis of this type cannot be disregarded by investors entirely, for some automated trading uses the indicators of this type of analysis.

The rate of return is known in the analysis as the rate of change (Murphy 1995). The rate of change is one of the best known indicators of the technical analysis. It can be calculated at any moment in which the price of the financial instrument analyzed is known. It is given by the following formula:

$$ROC(t,k) = \frac{p_t}{p_{t-k}} - 1 \qquad (1)$$

This is an indicator calculated at moment t, at a distance k. It speaks about a relative change in price from moment t-k until moment t. These are usually closing prices in individual time intervals, with this indicator being frequently expressed as percentage.

2. Black-Scholes model and Ito process

A very similar model of stock behavior is present in the Black-Scholes model for option pricing. The considerations will start with the analysis of a certain problem related to the function of two real variables. At first, the definition of Wiener process will be provided (Lipcer, Sziriajew 1981). Wiener process is a process fulfilling the following conditions:

$(i)\ \ P\,(W_0 = 0) = 1$

$(ii)\ for\ any\ s < t \le u < w\colon W_t - W_s\ i\ W_w - W_u\ are\ independent$

$(iii)\ \ W_t - W_s \sim N(0, \sqrt{t-s}\,)$.

Supposing that function F is differentiable (in the theory of the function of two variables, this

implies the existence of the continuity of the first-order partial derivatives), it is possible to calculate its complete differential. Its form (Hull 1997) is given
by formula (2):

$$dF(x,t) = F'_x dx + F'_t dt \qquad (2)$$

In practice (for example while estimating measure errors) a discrete version of equation (2) is used:

$$F(x+\Delta x, t+\Delta t) - F(x,t) \cong F'_x \Delta x + F'_t \Delta t \qquad (3)$$

The price process in the Black-Scholes model is a so called Ito process (Schuss 1989) given by:

$$dx = a(x,t)dt + b(x,t)dW, \qquad (4)$$

where W is Wiener process. Variables x and t are independent. Secondly, what needs to be clarified is the fact that differential dW does not exist in the formal sense. This situation calls for the application of a special definition of derivative for a function of unbounded variation.

A more precise version of formula (3) is obtained in that function F is expanded into Taylor series around the point (x,t):

$$F(x+\Delta x, t+\Delta t) - F(x,y) \cong F'_x \Delta x + F'_t \Delta t + \frac{1}{2} F''_{xx} (\Delta x)^2 + \frac{1}{2} F''_{tt} (\Delta t)^2 + F''_{xt} \Delta x \Delta t \quad (5)$$

This is the outcome known from the classical differential of a function of two variables. If $\Delta x \to 0$ *and* $\Delta t \to 0$ on the right-hand side of the equation (10) the second-order expressions may be dropped, which would bring the analyzed relation to form (3). Yet, in the case considered a clear correlation between x and t exists given by formula (4). A discrete version of this relation is expressed by formula (6):

$$\Delta x = a\Delta t + b\varepsilon\sqrt{\Delta t}\ , \qquad (6)$$

because variable $\Delta W = \varepsilon\sqrt{\Delta t}$ at a moment t has a standard deviation equal to $\sqrt{\Delta t}$, where ε is a random variable normally distributed.

It turns out that the expression $\frac{1}{2} F''_{xx} (\Delta x)^2$ from formula (5) may not be omitted because according to (6):

$$(\Delta x)^2 = a^2 (\Delta t)^2 + 2ab\Delta t\sqrt{\Delta t} + b^2 \varepsilon^2 \Delta t\ .$$

Moving on to limit $\Delta t \to 0$ in the expression above, one can see that it is possible to drop the higher- than-1-order expressions, obtaining roughly $(\Delta x)^2 \cong b^2 \varepsilon^2 \Delta t$. As
$E(\varepsilon^2) = 1$, so $E(\varepsilon^2 \Delta t\ )$=$\Delta t$.

Furthermore, $VAR(\varepsilon^2 \Delta t) = c\ (\Delta t)^2$, for a certain constant c. This means that for small Δt the variation approaches zero, which implies that the limit of $\varepsilon^2 \Delta t$ must be equal to its expected value, that is, $\Delta t,$ and it is a non-stochastic value. Now, moving in expression (5) to a complete differential (limit at $\Delta x \to 0 \ and \ \Delta t \to 0$) one will obtain the known expression, it being the thesis of Ito's lemma

$$dF = F_x' \mathrm{d}x + F_t' \mathrm{d}t + \frac{1}{2} F_{xx}'' b^2 dt \qquad (7)$$

Expression (7) differs from the complete differential, as it is known from mathematical analysis, in its last component, on the right-hand side of equation (6) of the expression defining Ito process, one will obtain the function F differential dependent on dt and dW.

$$dF = F_x'(a(x,t)dt + b(x,t)dW) + F_t' \mathrm{d}t + \frac{1}{2} F_{xx}'' b^2 dt\,, \qquad (8)$$

After regrouping the expressions in equation (8), one will obtain the final form of the differential of function F (a pair of arguments (x,t) was dropped):

$$dF = \left(F_x' a + F_t' + \frac{1}{2} F_{xx}'' b^2\right) dt + F_x' b dW \qquad (9)$$

Formula (9) explains that a drift rate is equal here to $F_x' a + F_t' + \frac{1}{2} F_{xx}'' b^2$ and the amount characterizing risk (an equivalent of standard deviation) is $F_x' b$. Formula (9) also explains why the drift rate is dependent, too, on the amount characterizing risk. The above derivation (based on the arguments presented in Hull 1997) allowed for avoiding the difficult problem involved in the definition of stochastic integrals. The application of the Ito process and Ito's lemma leads to a number of interesting results not only for derivative instruments.

3. A geometric Brownian motion

Assuming that the volatility of stocks is described by a geometric Brownian motion (Weron, Weron 1999):

$$dX = mXdt + \sigma X dW_t$$

or else $\frac{dX}{X} = mdt + \sigma dW_t$

It can be proven that function $S_t = S_0 \exp((m - \frac{\sigma^2}{2})t + \sigma W_t)$ is the solution of the equation describing the geometric Brownian motion. Let $X_t = at + bW_t = \left(m - \frac{\sigma^2}{2}\right) t + \sigma W_t$, that is,

$S_t = S_0 \exp(X_t)$. In order to prove the mentioned theorem, one should use Ito's lemma assuming that $F(x) = S_0 \exp(x)$. In the analyzed case $F'(x) = S_0 \exp(x)$ and $F''(x) = S_0 \exp(x)$. Therefore, according to Ito's lemma

$$dS_t = S_0 \exp(X_t)\, adt + S_0 \exp(X_t)\, bdW_t + \frac{1}{2} b^2 S_0 \exp(X_t) =$$

$$S_t\left(m - \frac{\sigma^2}{2}\right)dt + S_t \sigma dW_t + \frac{1}{2}\sigma^2 S_t dt = mS_t dt + \sigma S_t dW_t$$

After applying the similar reasoning to moments t and T (instead of 0 and t) and using the properties of Wiener process, one obtains:

$$Y = \ln\left(\frac{S_T}{S_t}\right) \sim N\left\{\left(m - \frac{\sigma^2}{2}\right)(T-t), \sigma\sqrt{T-t}\right\} \quad (10)$$

Putting in place of t numbers t_i starting with zero, and in place of T numbers $t_{i+1} > t_i$, while assuming that $t_{i+1} - t_i = \Delta t = 1/K$, (K denotes the number of trading days per year) one obtains a string of variables on non-overlapping intervals:

$(\ln\left(\frac{S_{t_1}}{S_0}\right), \ln\left(\frac{S_{t_2}}{S_{t_1}}\right), \ln\left(\frac{S_{t_3}}{S_{t_2}}\right), \dots, \ln\left(\frac{S_{t_N}}{S_{t_{N-1}}}\right) \dots)$.

According to formula (10), this is a string of random variables distributed in the same way:

$$Y_i = \ln\left(\frac{S_{t_i}}{S_{t_{i-1}}}\right) \sim \; N\left\{\left(m - \frac{\sigma^2}{2}\right)\Delta t, \sigma\sqrt{\Delta t}\right\}$$

Given that the intervals do not intersect, the variables are independent. This arises from the fact that price in the stochastic sense is dependent only on the current value of Wiener process. Thus, the price logarithm at moment t_i is such that:

$$\ln(S_{t_i}) = \left(m - \frac{\sigma^2}{2}\right)t_i + \sigma W_{t_i}$$

A similar relation occurs for argument t_{i-1}. After considering the difference of these two expressions, one obtains the difference:

$$Y_i = \ln\left(\frac{S_{t_i}}{S_{t_{i-1}}}\right) = \left(m - \frac{\sigma^2}{2}\right)(t_i - t_{i-1}) + \sigma(W_{t_i} - W_{t_{i-1}}). \quad (11)$$

The construction of the intervals using points t_i assumed that they would not overlap and so, the independence of variables Y_i comes as a consequence of postulate (ii) from the definition of Wiener process.

4. Order statistics

For the variables analyzed which are distributed in the same way and are independent (further on, the possibility of departure from this assumption will be given) it is possible to use the simplest version of the theorems on order statistics. These theorems will be used to construct statistical tests investigating the trend on the stock market analyzed here. Selected definitions and facts are presented below within the scope of limit theorems for order statistics.

The key theorem in the theory of order statistics is the one about extreme types. The concept of max-stable distribution functions is of importance. A distribution function G is stable if there exist constants $a_n > 0$ and b_n such that for every $G^n(a_n x + b_n) = G(x)$. A power distribution function is also a distribution function. For max-stable distribution functions, the power differs from the initial distribution function only in location parameters. Multiplication of distribution functions is an operation related to searching for the maximum distribution with respect to random variables. The distribution of a random variable $M_n = \max(X_1, X_2, \dots, X_n)$ is a max-stable distribution, assuming that random variables $X_1, X_2, \dots, X_n$ are independent and are distributed in the same way wiht this distribution being of the appropriate type.

A theorem which is the consequence of the theorem on extreme types is presented below. Here one uses the assumption on the normality of random variables analyzed (Leadbetter et al. 1986). It is presented as Theorem 1: If (X_n) is a string of independent random variables normally (in a standard way) distributed, then the distribution of M_n is max-stable and the asymptotic distribution of random variable $M_n = \max(X_1, X_2, \dots, X_n)$ fulfills for constants $a_n > 0$, and b_n (when $n \to \infty$) the relation

$$P(a_n(M_n - b_n) \leq x) \to \exp(-e^{-x}).$$

Numerical sequences $a_n > 0$ and b_n are given by

$$a_n = (2lnn)^{1/2}$$

$$b_n = a_n - \frac{lnlnn + \ln(4\pi)}{2a_n}$$

If one considers the random variables

$$Y_i = \ln\left(\frac{S_{t_i}}{S_{t_{i-1}}}\right) \sim \ N\left\{\left(m - \frac{\sigma^2}{2}\right)\Delta t, \sigma\sqrt{\Delta t}\right\}$$

it is easy to notice that they fulfill the assumptions of theorem 1. Therefore the theorem can be applied provided that parameters m and σ^2 have been estimated before. The relationship between the variables from the rate of change known from the technical analysis (Czekała 2001) results from an approximate equation $\ln(1+x) \approx x$. In the case in question this implies that

$$\ln\left(\frac{S_{t_i}}{S_{t_{i-1}}}\right) = \ln(1 + \left(\frac{S_{t_i}}{S_{t_{i-1}}} - 1\right)) \approx \frac{S_{t_i}}{S_{t_{i-1}}} - 1 = \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}}$$

The last expression in the equation above is equal to the rate of return. It is on its basis that the decisions whether to buy or sell securities are made. Practitioners tend to apply intuitive levels at which the asset analyzed is overbought (oversold). A large value of the indicator is interpreted as a signal to sell. A low (negative) values, on the other hand, are supposed to signal buying (Czekała 1998). In numerous algorithms for automated trading it is suggested that the level of +10% is a signal for selling within a week. The level -10% is to signal buying. Here, it is presupposed that the facts resulting from the fundamental analysis exert no impact on stock pricing for speculation purposes, or the assumption is also that the financial data on quoted enterprises are accounted for in stock pricing (partial market efficiency). Even with this assumption, the levels denoting buying or selling ought to depend at least on the volatility of rates of return. In this sub-chapter, a proposal fulfilling this postulate is presented.

As an example, the rate of change will be considered for non-overlapping intervals. One can then assume that the rates of return under analysis are independent. Variables $Y_i = \ln\left(\frac{S_{t_i}}{S_{t_{i-1}}}\right)$ should be in the first place standardized. This means that the parameters have to be estimated based on a sample that does not cover the period in which the buying and selling signals are generated. In this way one obtains standardized values for the rate of change (logarithmic version):

$$\widetilde{Y}_i = \frac{Y_i - (m - \frac{\sigma^2}{2})}{\sigma} \quad (12)$$

Values m and σ are usually unknown (the exception here are simulation studies – where by definition they can be known) and must be estimated based on a sample. In this case the evaluation values will be replaced by the population parameters present in formula (12).

5. An empirical example – the rates of return analysis[1]

In the first two columns of Table 1 the data on the share analyzed within a trial period (30 weeks) are presented. On the basis of the data from this period parameters m and σ^2 were estimated.

Table 1. KGHM share prices

date	price	date	price	date	price
10-April-15	114.35	11-Sept-15	76.39	12-Feb-16	60.48
17-Aprik-15	113.12	18-Sept-15	81.79	19-Feb-16	59.06
24-April-15	112.93	25-Sept-15	85.07	26-Feb-16	67.1
01-May-15	113.32	02-Oct-15	79.44	04-March-16	66.12
08-May-15	120.14	09-Oct-15	78.86	11-March-16	74.44
15-May-15	121.38	16-Oct-15	94.07	18-March-16	71.9
22-May-15	122.33	23-Oct-15	90.31	25-March-16	73.95
29-May-15	119.91	30-Oct-15	97.94	01-April-16	74.78
05-June-15	113.03	06-Nov-15	88.4	08-April-16	73.26
12-June-15	110.83	13-Nov-15	87.67	15-April-16	65.87
19-June-15	110.19	20-Nov-15	75.71	22-April-16	68.34
26-June-15	108.23	27-Nov-15	72.48	29-April-16	73.71
03-July-15	105.78	04-Dec-15	71.5	06-May-16	72.75
10-July-15	101.67	11-Dec-15	65.63	13-May-16	66.14
17-July-15	102.36	18-Dec-15	60.61	20-May-16	63.18
24-July-15	100.01	25-Dec-15	59.06	27-May-16	60.93
31-July-15	88.15	01-Jan-16	61.24	03-June-16	61.71
07-Aug-15	92.07	08-Jan-16	58.77	10-June-16	59.87
14-Aug-15	91.48	15-Jan-16	51.37	17-June-16	58.72
21-Aug-15	89.62	22-Jan-16	50.44	24-June-16	59.96
28-Aug-15	78.36	29-Jan-16	54.64	01-July-16	60.78
04-Sept-15	77.87	05-Feb-16	55.83	08-July-16	65,92

Source: Stooq (2016).

Table 2. Auxiliary calculations

st. rate of	an	bn	Mn	X	p-value
return	2.620682	1.900889			
-2.28956	2.632769	1.912975	-2.28956	-11.0643	1
-0.55842	2.644431	1.924637	-0.55842	-6.56627	1

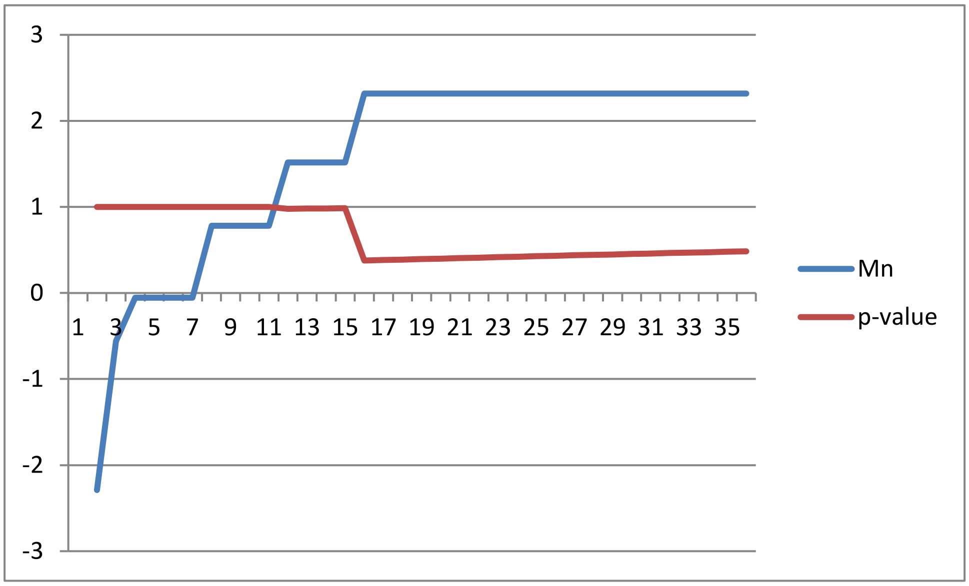
[1] (Czekała, Bukietyńska 2017).

-0.05477	2.655696	1.935902	-0.05477	-5.28663	1
-1.26493	2.666589	1.946795	-0.05477	-5.33736	1
-1.16262	2.677132	1.957339	-0.05477	-5.38669	1
-0.26124	2.687347	1.967554	-0.05477	-5.43469	1
0.782665	2.697253	1.977459	0.782665	-3.22266	1
-0.51759	2.706866	1.987072	0.782665	-3.26017	1
-2.08646	2.716203	1.99641	0.782665	-3.29678	1
-0.13298	2.725279	2.005485	0.782665	-3.33253	1
1.517229	2.734107	2.014313	1.517229	-1.35908	0.979608
0.53574	2.742699	2.022906	1.517229	-1.38692	0.98173
1.517559	2.751069	2.031275	1.517559	-1.41327	0.983582
-0.22518	2.759225	2.039432	1.517559	-1.43997	0.985309
2.317522	2.76718	2.047386	2.317522	0.747515	0.377208
-0.07324	2.774941	2.055147	2.317522	0.728075	0.38297
2.164546	2.782517	2.062724	2.317522	0.708981	0.388687
-0.40923	2.789918	2.070124	2.317522	0.69022	0.394358
0.646051	2.79715	2.077356	2.317522	0.67178	0.399984
0.361335	2.80422	2.084427	2.317522	0.653651	0.405565
-0.17104	2.811136	2.091343	2.317522	0.635821	0.411101
-1.61205	2.817904	2.098111	2.317522	0.61828	0.416593
0.792162	2.82453	2.104736	2.317522	0.60102	0.422041
1.44436	2.831019	2.111225	2.317522	0.584031	0.427445
-0.04631	2.837376	2.117583	2.317522	0.567303	0.432805
-1.42601	2.843607	2.123814	2.317522	0.55083	0.438122
-0.59514	2.849717	2.129923	2.317522	0.534603	0.443396
-0.43518	2.855709	2.135916	2.317522	0.518615	0.448627
0.387521	2.861589	2.141795	2.317522	0.502858	0.453816
-0.33454	2.867359	2.147565	2.317522	0.487327	0.458962
-0.15188	2.873024	2.153231	2.317522	0.472013	0.464067
0.524859	2.878588	2.158795	2.317522	0.456911	0.469129
0.402012	2.884054	2.16426	2.317522	0.442015	0.47415
1.537373	2.889425	2.169631	2.317522	0.42732	0.47913
2.317522	2.894704	2.17491	2.317522	0.412819	0.484069

Source: Author's own calculation.

The estimation was up-dated and always referred to the last 30 sessions before the session for which a possible signal was to occur for selling or a warning about the lack of stability. The calculations are presented in Table 2. Figure 1 presents the values of standardized maximum and p-values.

Figure 1. P-value and standardized max-values



Source: self-reported data.

The last column of the table in question contains p-values for double-standardized maximum value. A low value (comparable with the usual levels of significance) could attest to the occurrence of an event eliminating the supposition as to the stability of parameters. The language of the technical analysis would interpret such event as a signal to sell.

None of the values presented in the last column is low compared to the levels of significance usually applied. That is why the rates of return observed do not provide a proof of instability.

6. Conclusion

The assumptions adopted in this work about the behavior of stock prices stem from the classic models of derivative pricing. In drawing on these assumptions – using the order statistics theory – an asymptotic distribution of standardized rates of return was presented. The findings can be applied in testing the stability of the models of financial instrument pricing as well as in the technical analysis. In the latter case, the max distribution can be used to generate signals to sell.

Employing a similar method, the method outlined here can be used in generating signals to buy. Instability of rates of return can also be obtained as a conclusion drawn from the analysis of statistics –minimum. To this end, it suffices to notice that

$$min(Y_1, Y_2, \dots, Y_n) = -max(-Y_1, -Y_2, \dots, -Y_n).$$

While knowing the max-distribution it is not that difficult to determine the minimum distribution, and, on its basis, to either construct a stability test or a model generating signals to buy. These signals are crucial from the perspective of the effectiveness of investing while also increasing potential profits. A correct reading of these signals is extremely important for individual and professional investors.

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