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ISSN electronic version 2353 - 9119
Central and Eastern European Journal
of Management and Economics
Vol. 5, No. 3, 277-289, September 2017

The test of inversion in the analysis of investment funds

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Abstract:

Aim: The aim of the paper is to evaluate the quality of investing at OFE (open-ended pension funds) based on the data spanning the period of 2002-2010.

Design/Research method: In the study, the test of inversion was used as a measure of dependence. In the classification of the funds Sharpe measure was adopted.

Conclusions/Findings:. Contrary to common expectations, the ranking obtained based on Sharpe measure showed randomness in the ordering of funds, with the period of 2009-2010 being the only exception. These years preceded the first "reform" of OFE.

Originality/Value of the article: For the first time, apart from testing the hypothesis on ranking randomness, an analysis of Type II error was presented, that is, an error consisting in accepting null hypothesis (on randomness) despite its being false.

Keywords: Kendall's coefficient, Sharpe measure, test power, test of inversion
JEL: C12, C46

1. Introduction

The aim of the paper is to evaluate Open-ended Pension Funds based on historical data using the test of inversion. In examining the effectiveness of investments using Sharpe measure (Wilimowska, Wilimowski 2002), one obtains a specific order. Investors often see the order given as a guideline for future investments. Kendall's coefficient is a well-known coefficient

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http://dx.doi.org/10.29015/ceejme.633

used in examining rank correlation. As a measure of dependence, it is used for any sample size. Its distribution (with the exception of asymptotic distribution) is less likely to be used because of a rather difficult analytic form of the statistics used in testing the relevant hypotheses. In this work, the test of inversion will be applied which is a variant of the test based on Kendall rank correlation. For a moderate size of a sample, it appears more convenient to consider the number of inversions. The number is equal to the number of discordant pairs (in the sense outlined below) for continuously distributed variables (tied pairs are not possible then). It turns out that the language of inversion is often more convenient. This becomes especially visible for the Type II error analysis (Barra 1982).

It will be this variant of Kendall test based on inversions that will be used as a test supporting the study on the effectiveness of investments of the well-known group of funds. The numerous "reforms" of pension schemes (Oręziak 2010; Bukietyńska, Czekała 2011: 23-34) require the considerations to be limited; they begin on the date the schemes were founded and continue until 2010. In the individual years in question, Sharpe measure was used for the classification of the funds. In accordance with the assumptions underlying the financial theory, this measure should be employed in the evaluation of the portfolio quality management, yet, it also should include an element of forecasting future performance. The fact that the reforms were carried out after 2010 had no impact on the results of the analyses. It was interesting to look at whether it was possible to apply this measure to show the funds which were most likely to be ranked at the top.

2. Kendall's coefficient

Kendall's tau coefficient (Magiers 2002) is used to describe correlations between ordinal variables. In order to calculate Kendall's tau, all observations from the sample should be combined in all possible pairs and divided into three categories. Concordant pairs – variables which are in the first observation either bigger than in the second one, or both are smaller; the number of such pairs will be denoted as P_z. Discordant pairs – variables change inversely, that is, one of them is bigger for a given observation in the pair for which the second one is smaller; the number of such pairs will be denoted as P_n. Tied pairs – in both observations, one of the variables has the same values, the number of such pairs - P_w. Kendall's τ estimator can be calculated from the formula:

$$\tau = \frac{P_z - P_n}{P_z + P_n + P_w}$$

The coefficient is within the interval (-1, 1).

Since $P_z + P_n + P_w = \binom{n}{2} = \frac{n(n-1)}{2}$

then

$$\tau = 2\frac{P_z - P_n}{n(n-1)}$$

where:

n –. a sample size

P_z - the number of concordant pairs

P_n - the number of discordant pairs

3. Inversions

A convenient tool in the analysis of variables on the ordinal scale are permutations. A permutation is a function rearranging a set of natural numbers $\{1, 2, \dots, n\}$ onto itself. The observations of any real random variable can be ordered according to the natural order if there are no values equal to one another. This occurs with the assumption that the random variable in question is continuous.

Let

$$N_n = \frac{n(n-1)}{2}$$

be the maximum number of inversions in a permutation of n arguments.

Let $\begin{matrix} N_n \\ k \end{matrix}$ be the number of permutation having exactly k inversions.

If $N_1 = 1$, then from the definition $\left\{\begin{matrix} N_1 \\ 0 \end{matrix}\right\} = 0$

For $N_2 = 2$, is $\left\{\begin{matrix} N_2 \\ 0 \end{matrix}\right\} = 1$ and $\left\{\begin{matrix} N_2 \\ 1 \end{matrix}\right\} = 1$.

For $N_3 = 3$ is $\left\{\begin{matrix} N_3 \\ 0 \end{matrix}\right\} = \left\{\begin{matrix} 3 \\ 0 \end{matrix}\right\} = 1, \left\{\begin{matrix} N_3 \\ 1 \end{matrix}\right\} = \left\{\begin{matrix} 3 \\ 1 \end{matrix}\right\} = 2, \left\{\begin{matrix} N_3 \\ 2 \end{matrix}\right\} = \left\{\begin{matrix} 3 \\ 2 \end{matrix}\right\} = 2, \left\{\begin{matrix} N_3 \\ 3 \end{matrix}\right\} = \left\{\begin{matrix} 3 \\ 3 \end{matrix}\right\} = 1$

In a similar way, for $N_4 = 6$:

$$\left\{\begin{matrix} N_4 \\ 0 \end{matrix}\right\} = 1, \left\{\begin{matrix} N_4 \\ 1 \end{matrix}\right\} = 3, \left\{\begin{matrix} N_4 \\ 2 \end{matrix}\right\} = 5, \left\{\begin{matrix} N_4 \\ 3 \end{matrix}\right\} = 6,$$

$$\left\{\begin{matrix} N_4 \\ 4 \end{matrix}\right\} = 5, \left\{\begin{matrix} N_4 \\ 5 \end{matrix}\right\} = 3, \left\{\begin{matrix} N_4 \\ 6 \end{matrix}\right\} = 1$$

In general :

$$\left\{ {N_n \atop k} \right\} = \sum_{i=\max(0,k-n+1)}^{k} \left\{ {N_{n-1} \atop i} \right\} \qquad (1)$$

The sequence under consideration is well-known from The On-Line Encyclopedia of Integer Sequences as A008302 sequence.

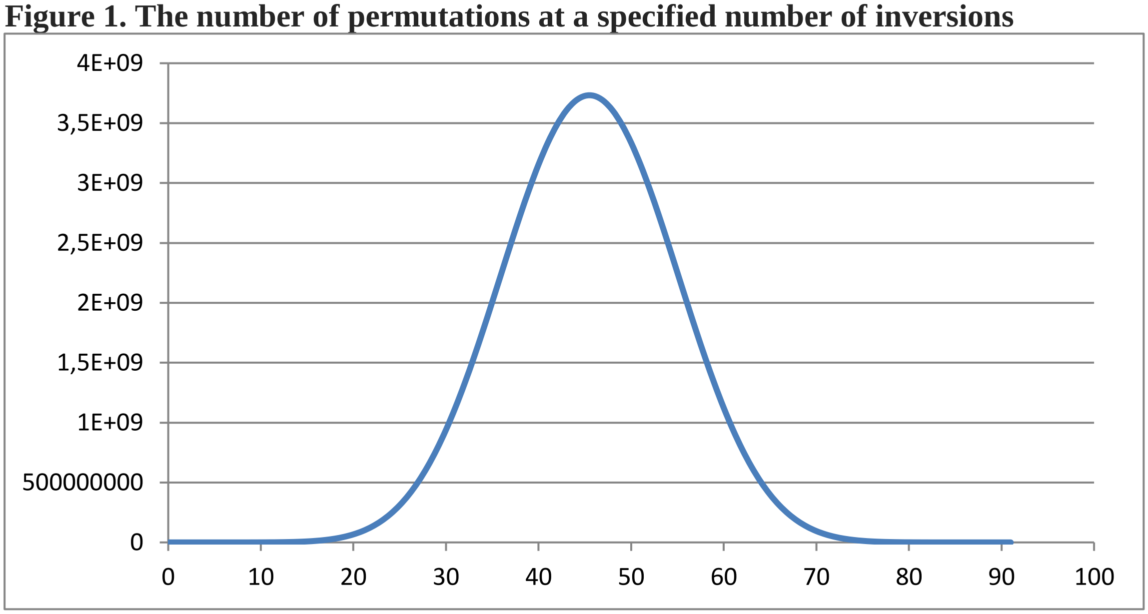
Table 1. A008302 sequence for the selected n and number of inversions

n/Inversions	0	1	2	3	4	5	6	7	8	9
1	1									
2	1	1								
3	1	2	2	1						
4	1	3	5	6	5	3	1			
5	1	4	9	15	20	22	20	15	9	4
6	1	5	14	29	49	71	90	101	101	90
7	1	6	20	49	98	169	259	359	455	531
8	1	7	27	76	174	343	602	961	1415	1940
9	1	8	35	111	285	628	1230	2191	3606	5545
10	1	9	44	155	440	1068	2298	4489	8095	13640
11	1	10	54	209	649	1717	4015	8504	16599	30239

Source: Self-reported data (Bukietyńska A., Czekała M. 2017)

For n=14, the relevant inversion numbers are presented in Figure 1.

Figure 1. The number of permutations at a specified number of inversions



Source: Author's own calculations.

4. The application of the test of inversion in the evaluation of OFE funds

For all the 14 OFE funds the rates of return were calculated for the period of 2002-2010. The next step involved the calculation of Sharpe coefficient (Haugen 1996) which unfortunately turned out to be negative for 2008 and therefore the rates of return were used there.

Sharpe coefficient was calculated from the formula:

$$S = \frac{R_j - R_f}{\sigma R_j},$$

where:

R_j – the fund's average return over the period studied

R_f – the average risk-free rate over the period studied, in this case WIBOR 1m

σR_j – standard deviation of the rates of return over a given period

Table 2. Sharpe coefficients over the period of 2002-2010 for 14 funds

YEAR	AEGON	Allianz	Amplico	Aviva	AXA	Generali	ING	Nordea	Pekao	PKO PB	Pocztylion	Polsat	PZU	Warta
2002	0.146	0.234	0.168	0.095	0.036	0.112	0.244	0.226	-0.129	0.293	0.027	0.067	0.183	0.035
2003	0.153	0.200	0.196	0.147	0.150	0.211	0.173	0.190	0.201	0.176	0.156	0.339	0.204	0.213
2004	0.543	0.478	0.577	0.529	0.731	0.598	0.477	0.493	0.728	0.605	0.549	0.494	0.581	0.721
2005	0.312	0.273	0.408	0.392	0.346	0.364	0.391	0.319	0.264	0.279	0.354	0.415	0.301	0.346
2006	0.437	0.530	0.448	0.456	0.453	0.570	0.440	0.413	0.616	0.446	0.517	0.667	0.472	0.472
2007	0.056	0.079	0.097	0.098	0.071	0.055	0.030	0.039	0.083	-0.030	0.007	-0.065	0.089	-0.007
2008	-0.608	-0.612	-0.631	-0.627	-0.589	-0.676	-0.578	-0.602	-0.672	-0.658	-0.594	-0.749	-0.599	-0.658
2009	0.371	0.351	0.334	0.290	0.339	0.373	0.310	0.318	0.342	0.372	0.335	0.496	0.288	0.315
2010	0.302	0.336	0.386	0.346	0.336	0.284	0.351	0.359	0.306	0.363	0.343	0.278	0.332	0.348

Source: Author's own calculations

Sharpe measure is precisely what provides the basis for creating the funds ranking (Reilly, Brown 2001). This method is usually employed in the evaluation of the investment quality for investment funds. In order to calculate the number of inversions, the funds were numbered depending on their rank. Which was then followed by the calculation of the number of inversions. In this way all the years were string-like compared. In Table 3 some of the calculations for 2008 and 2009 are presented.

Table 3. Sharpe ratios comparison for 2008 and 2009

Sharpe	2008	no	m2008	m2009	Sharp	2009	no
-0.01022	Allianz Polska	2	1	5	0.495678	Polsat	12
-0.01074	AXA	5	2	7	0.372974	Generali	6
-0.01077	Pocztylion	11	3	8	0.371857	PKO PB	10
-0.01111	Nordea	8	4	10	0.370536	AEGON	1
-0.0114	AEGON	1	5	4	0.350824	Allianz Polska	2
-0.0115	Generali	6	6	2	0.341838	Pekao	9
-0.01183	PKO PB	10	7	3	0.339444	AXA	5
-0.01186	Amplico OFE	3	8	9	0.335289	Pocztylion	11
-0.01220	PZU Złota Jesień	13	9	14	0.333830	Amplico OFE	3
-0.01234	Warta	14	10	11	0.318366	Nordea	8
-0.01253	ING	7	11	12	0.315157	Warta	14
-0.01259	Pekao	9	12	6	0.310295	ING	7
-0.01326	Aviva	4	13	13	0.290172	Aviva	4
-0.01587	Polsat	12	14	1	0.287749	PZU Złota Jesień	13

Source: Author's own calculations.

Table 4 contains analogous – together with the ranking – calculations for 2009 (repeating the calculations from Table 3) and 2010.

Table 4. Sharpe ratios comparison for 2009 and 2010

Sharpe	2009	no	m2009	m2010	Sharp	2010	no
0.495678	Polsat	12	1	14	0.385618	Amplico OFE	3
0.372974	Generali	6	2	13	0.363325	PKO PB Bankowy	10
0.371857	PKO PB Bankowy	10	3	2	0.359113	Nordea	8
0.370536	AEGON	1	4	12	0.351476	ING	7
0.350824	Allianz Polska	2	5	9	0.348023	Warta	14
0.341838	Pekao	9	6	11	0.346011	Aviva	4
0.339444	AXA	5	7	8	0.343285	Pocztylion	11
0.335289	Pocztylion	11	8	7	0.336281	AXA	5
0.333830	Amplico OFE	3	9	1	0.336131	Allianz Polska	2
0.318366	Nordea	8	10	3	0.332437	PZU Złota Jesień	13
0.315157	Warta	14	11	5	0.306408	Pekao	9
0.310295	ING	7	12	4	0.301738	AEGON	1
0.290172	Aviva	4	13	6	0.283659	Generali	6
0.287749	PZU Złota Jesień	13	14	10	0.278029	Polsat	12

Source: Author's own calculations.

Next, the number of inversions and probabilities were calculated.
The maximum number of inversions was calculated from the formula $N_n = \frac{n(n-1)}{2} = \frac{14(14-1)}{2} 91$

Table 5. The number of inversions over the period of 2002-2006

m2002	1	2	3	4	5	6	7	8	9	10	11	12	13	14				
m2003	9	10	6	8	4	7	12	3	14	1	13	2	11	5				
															inversion	max	p	average
l.inw	8	8	5	6	3	4	5	2	5	0	3	0	1	0	**50**	**91**	**0.549**	**45.5**
m2003	1	2	3	4	5	6	7	8	9	10	11	12	13	14				
m2004	11	3	5	6	2	13	7	12	4	14	8	9	1	10				
															inversion	max	P	Average
l.inw	10	2	3	3	1	7	2	5	1	4	1	1	0	0	**40**	**91**	**0.44**	**45.5**
m2004	1	2	3	4	5	6	7	8	9	10	11	12	13	14				
m2005	7	14	8	12	5	11	2	6	10	3	1	9	13	4				
															inversion	max	P	Average
l.inw	6	0	6	9	4	7	1	3	4	1	0	1	0	0	**42**	**91**	**0.462**	**45.5**
m2005	1	2	3	4	5	6	7	8	9	10	11	12	13	14				
m2006	1	10	8	12	3	5	9	7	14	13	6	11	4	2				
															inversion	max	P	Average
l.inw	0	8	6	8	1	2	4	3	5	4	2	2	1	0	**46**	**91**	**0.505**	**45.5**

Source: Author's own calculations

In Table 5, the numbers of inversions calculated did not differ much from the average number of inversions which was at 45.5. This does not provide yet a basis for drawing conclusions.

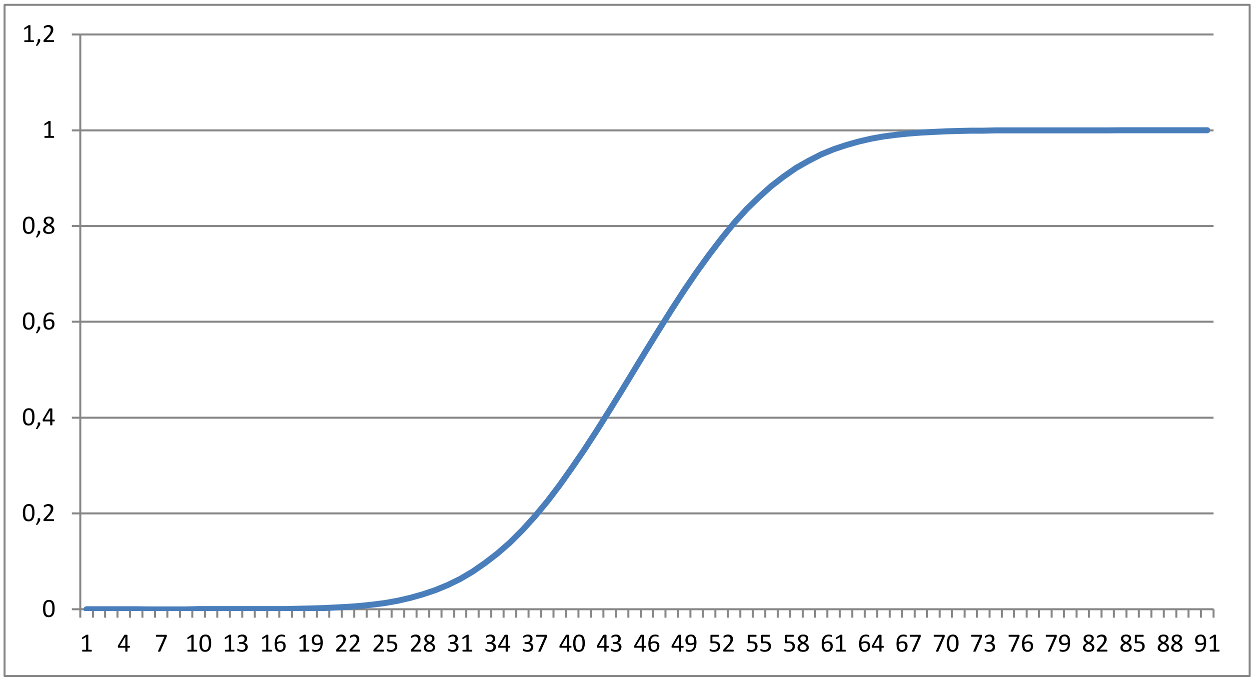
Table 6. The number of inversions over the period of 2006-2010

m2006	1	2	3	4	5	6	7	8	9	10	11	12	13	14				
m2007	14	1	8	5	11	4	12	2	6	3	13	10	7	9				
															inversion	max	p	average
l.inw	13	0	6	3	7	2	6	0	1	0	3	2	0	0	**43**	**91**	**0.473**	**45.5**
m2007	1	2	3	4	5	6	7	8	9	10	11	12	13	14				
m2008	12	13	8	9	1	2	5	6	4	11	3	10	7	14				
															inversion	max	P	Average
l.inw	11	11	7	7	0	0	2	2	1	3	0	1	0	0	**45**	**91**	**0.495**	**45.5**
m2008	1	2	3	4	5	6	7	8	9	10	11	12	13	14				
m2009	5	7	8	10	4	2	3	9	14	11	12	6	13	1				
															inversion	max	P	Average
l.inw	4	5	5	6	3	1	1	2	5	2	2	1	1	0	**38**	**91**	**0.418**	**45.5**
m2009	1	2	3	4	5	6	7	8	9	10	11	12	13	14				
m2010	14	13	2	12	9	11	8	7	1	3	5	4	6	10				
															inversion	max	P	Average
l.inw	13	12	1	10	7	8	6	5	0	0	1	0	0	0	**63**	**91**	**0.692**	**45.5**

Source: Author's own calculations

In Table 6, in the majority of cases the numbers of inversions calculated did not differ much from the average inversion number at 45.5. However, over the period of 2009 and 2010, the number of inversions observed differed rather significantly from the average that was at 45.5. Based on formula (1) a distribution function for the number of inversions was determined.

Figure 2. The distribution function for inversions



Source: Author's own study.

The hypothesis for 2008/2009 is tested:

$$H_0: PI = 0.5$$

$$H_1: PI < 0.5$$

The number of inversions I=38

$$P(I \leq 38/H_0) = 0.225$$

For 2009/2010

$$H_0: PI = 0.5$$

$$H_1: PI > 0.5$$

The number of inversions I=63

$$P(I \geq 63/H_0) = 0.018$$

For the significance levels commonly employed (in this case $\alpha = 0.05$) one can notice that in the first case there are no grounds for rejecting H_0, whereas in the second case – H_0 is rejected.

The results covering all the years studied are presented in Table 7.

Table 7. Hypothesis testing for the period of 2002-2010

years	number of Inversions	p-value	DECISION
2002/2003	50	0.705	accept H_0
2003/2004	40	0.295	accept H_0
2004/2005	42	0.374	accept H_0
2005/2006	46	0.543	accept H_0
2006/2007	43	0.415	accept H_0
2007/2008	45	0.500	accept H_0
2008/2009	38	0.225	accept H_0
2009/2010	63	0.018	reject H_0

Source: Author's own study.

The decisions presented in Table 7 in the vast majority of cases do not question the hypothesis tested. This means that Sharpe measure as a criterion for the quality of investment decisions does not have a forecasting value in the majority of cases. The arrangement of the funds analyzed appears to be incidental.

It turns out that the method analyzed together with the application of a precise distribution of the number of inversions may be used in the analysis of the test power. To this end the theorem proved in (Bukietyńska, Czekała 2017: 175-185) can be used.

5. Theorem

The distribution of the number of inversions at the inversion probability equal to p (and q=1-p) is expressed by the formula:

$$P(I_n = k) = \left(\left\{ {N_n \atop k} \right\} p^k q^{N_n-k}\right) \Big/ \sum_{k=0}^{N_n} \left\{ {N_n \atop k} \right\} p^k q^{N_n-k} = (p_{n,k} \cdot p^k q^{N_n-k}) / \sum_{k=0}^{N_n} p_{n,k} \cdot p^k q^{N_n-k}$$

In the above theorem, values $p_{n,k}$ denote the probability that k inversions will occur in a sequence of length n, while assuming that the probability of inversion is at p=0.5.

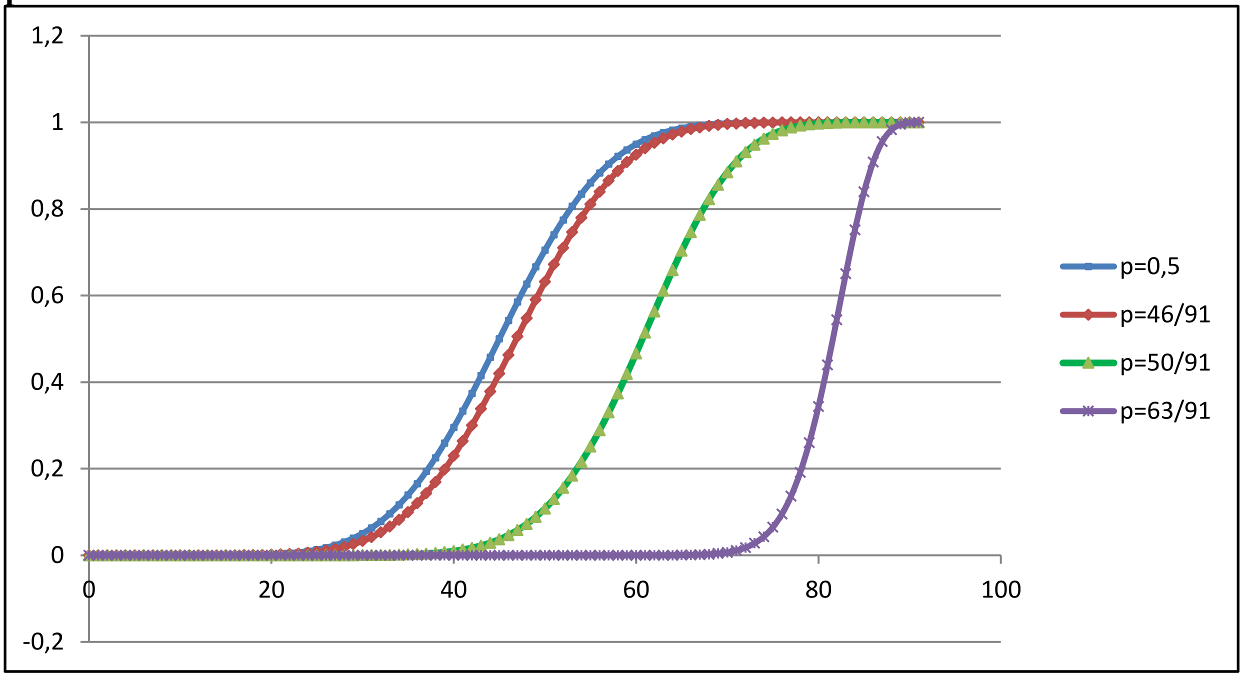
The distribution function of the number of inversions was calculated based on the above formula. This allows for the calculation of the probability that Type II error will be made for selected values p in an alternative hypothesis.

Table 8. The distribution function for the number of inversions for selected inversion probabilities

p/k	**0**	**1**	**2**	**3**	**4**	**5**	**6**
0.5	1.14707E-11	1.61E-10	1.19E-09	6.25E-09	2.59E-08	9.05E-08	2.76E-07
46/91	4.13556E-12	5.91E-11	4.48E-10	2.4E-09	1.01E-08	3.61E-08	1.13E-07
50/91	2.83217E-16	4.77E-15	4.27E-14	2.69E-13	1.34E-12	5.64E-12	2.07E-11
63/91	6.29862E-36	1.91E-34	3.06E-33	3.47E-32	3.12E-31	2.36E-30	1.56E-29
7	**8**	**9**	**10**	**11**	**12**	**13**	**14**
7.57E-07	1.9E-06	4.41E-06	9.6E-06	1.97E-05	3.86E-05	7.23E-05	0.00013
3.15E-07	8.05E-07	1.91E-06	4.24E-06	8.9E-06	1.78E-05	3.39E-05	6,22E-05
6.84E-11	2.06E-10	5.76E-10	1.51E-09	3.73E-09	8.77E-09	1.97E-08	4,27E-08
9.27E-29	5.04E-28	2.54E-27	1.2E-26	5.37E-26	2.28E-25	9.28E-25	3,63E-24
…	…	…	…	…	…	…	…
30	**31**	**32**	**33**	**34**	**35**	**36**	**37**
0.05051	0.063394	0.078582	0.096254	0.116556	0.13959	0.165406	0,193994
0.03297	0.042151	0.053214	0.066374	0.081827	0.099749	0.120283	0,143527
0.000301	0.00045	0.000665	0.000969	0.001396	0.001987	0.002795	0,003885
3.42E-16	9.27E-16	2.48E-15	6.54E-15	1.7E-14	4.39E-14	1.11E-13	2,8E-13
…	…	…	…	…	…	…	…
43	**44**	**45**	**46**	**47**	**48**	**49**	**50**
0.414955	0.457242	0.5	0.542758	0.585045	0.626406	0.666413	0,70468
0.338365	0.378465	0.419913	0.462281	0.505114	0.54794	0.590285	0,631689
0.022298	0.028768	0.036747	0.046477	0.058212	0.072209	0.088721	0,107981
5.51E-11	1.28E-10	2.93E-10	6.66E-10	1.49E-09	3.32E-09	7.29E-09	1,58E-08
…	…	…	…	…	…	…	…
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**
0.960272	0.969171	0.976412	0.982217	0.9868	0.990361	0.993081	0,995123
0.94019	0.952724	0.96315	0.971694	0.978589	0.984065	0.988341	0,991623
0.514529	0.562994	0.611084	0.658101	0.703367	0.746255	0.786216	0,822802
3.78E-05	7.13E-05	0.000132	0.000243	0.000439	0.000782	0.001372	0,002367
…	…	…	…	…	…	…	…
84	**85**	**86**	**87**	**88**	**89**	**90**	**91**
1	1	1	1	1	1	1	1
0.999999	1	1	1	1	1	1	1
0.99984	0.999937	0.999978	0.999994	0.999999	1	1	1
0.751575	0.839522	0.908288	0.955428	0.982702	0.995225	0.999296	1

Source: Author's own calculations.

Figure 3. The distribution function of the number of inversions for selected inversion probabilities



Source: self-reported data.

6. Type II error

In order to evaluate the power of the test, the probability of Type II error β is to be calculated. Type II error is to accept hypothesis $H_0$when it is false, which means that H_1 is the true hypothesis. These errors will be calculated for selected values of probabilities, that is, for $\frac{46}{91}$, $\frac{50}{91}$ and $\frac{63}{91}$.

$$\beta = P(accep\, H_0/H_1) = P\left(accept\, H_0/p = \frac{46}{91}\right) = P\left(I_{14} \geq 61/p = \frac{46}{91}\right) = 0.94$$

$$\beta = P(accep\, H_0/H_1) = P\left(accept\, H_0/p = \frac{50}{91}\right) = P\left(I_{14} \geq 61/p = \frac{50}{91}\right) = 0.515$$

$$\beta = P(accep\, H_0/H_1) = P\left(accept\, H_0/p = \frac{63}{91}\right) = P\left(I_{14} \geq 61/p = \frac{63}{91}\right) = 3.78 \cdot 10^{-5}$$

These probabilities were taken from Table 8 for 61 inversions. While using the theorem cited above, it is possible to create a complete version of this table.

7. Findings

The findings are two-fold. From the standpoint of the economic analysis, the fact that there are no reasons for rejecting null hypothesis attests to a complete randomness of the ranking. The financial results of the funds analyzed may change year by year with the probability at 0,5 (null hypothesis), with the period of 2009-2010 being the only exception (data in Table 7). In this case, however, the conclusion is even further – reaching. The probability of inversions that is above 0,5 (as the alternative hypothesis states) suggests that high ranking in one year makes a drop in ranking in the following year more likely.

The second kind of findings is strictly statistic in nature. The theorem cited allowed for calculating the probability that Type II error will be made for selected values adopted in the alternative hypothesis. The theorem and calculations conducted on its basis in Table 8 provide a useful tool for testing hypotheses on correlations in the case of an ordinal scale. Based on the theorem, it is possible to calculate the Type II error at any alternative. For the cases presented in the paper this probability was as high as 0,94, when $p = \frac{46}{91}$ was adopted in the alternative hypothesis. This last value differs slightly from the one proposed in the hypothesis tested. If $p = \frac{50}{91}$, a considerable increase in the test power was observed. The case that was most suggestive was when $p = \frac{63}{91}$; then the probability of Type II error takes on a very low value (Barra 1982).

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