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Simplification of decision problem structure by means of DEMATEL application

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Abstract:

Aim: Complex analysis results are often simplified for practical reasons. The choice of the simplification range for analysis results is critical for their reliability and the effects of their subsequent application. However, the need for the assessment of the simplification range assumed is often neglected. The same is true in the case of DEcision MAking Trial and Evaluation Laboratory (DEMATEL) technique application. The technique represents a popular tool for the identification of complex structures. DEMATEL enhancement is therefore proposed in the paper to avoid the excessive simplification of the results obtained through its application.

Design/Research method: The original DEMATEL procedure is modified to allow for determining the acceptable level of the simplification of its results.

Conclusions/Findings: The presented sample analysis reveals the feasibility of the modified DEMATEL procedure.

Originality/Value of the article: An important gap in DEMATEL theory is addressed in the paper. The gap may lead to incorrect outcomes with respect to further application of the results provided by DEMATEL application.

Keywords: structure, simplification, DEMATEL
JEL: C02, C44, C65

1. Introduction

The DEMATEL method was developed to identify cause-and-effect relations unfolding among the contemporary issues characterizing mankind (Fontela, Gabus 1976). In time, however, it became a universal tool for a decision-making analysis and valuable enhancement of other tools, too (Dytczak, Ginda 2015). Nowadays its most popular application involves identification

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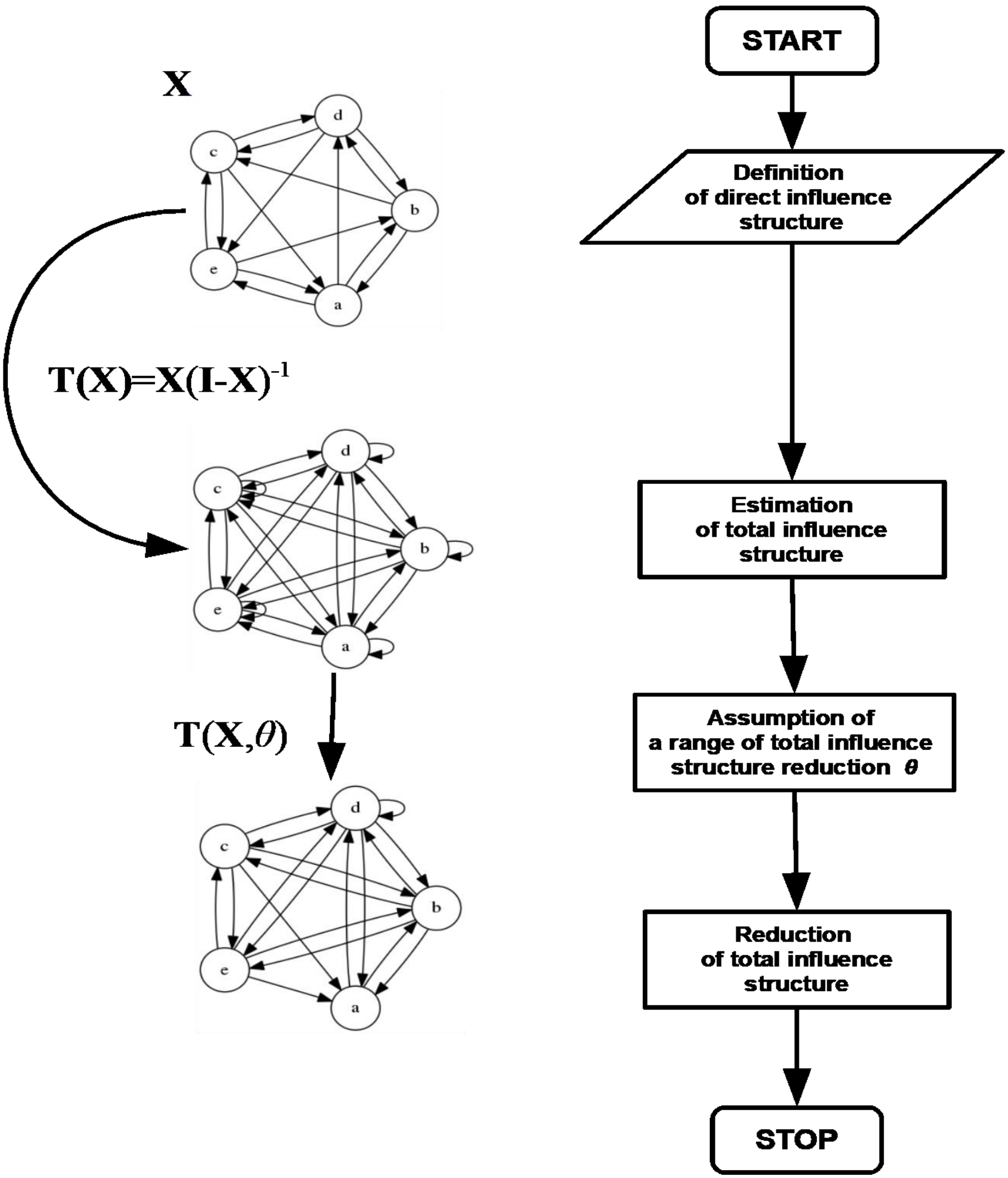
of influence structures among objects. In this context, the method provides an alternative for such tools as ISM (Warfield 1974), MICMAC (Duperrin, Godet 1973) and SEM (Goldberger 1972). Using the method we often obtain a picture of interactions that is complex and therefore rather unclear. That is why the method is simplified in that one abandons the weakest interactions. However, the range within which the influence structure is simplified is determined in a subjective way, which threatens the reliability of the results obtained through this method. The consequences of excessive simplification of structures may be acute in particular when they are used to feed data of other tools, e.g. a cognitive map (Kosko 1986), the analytic network process (ANP) (Saaty 1996), etc., whose calculation mechanisms are conducive to propagating and multiplying the effects coming from the application of a false influence structure of objects. Thus, seeking to ensure the reliability of results derived from the application of these tools, it is important to use an appropriate – not overly simplified – form of the influence structure. Therefore, in searching for this form, one should pay attention to assessing the consequences in terms of the choice of the structures under consideration. Yet, the methods currently applied and aimed at defining the simplification range do not allow that. The objective of the paper is therefore to enhance the DEMATEL method by accounting for the consequences arising from the choice of simplification range while searching for its appropriate form.

The paper is divided into the following sections. Section 2 is concerned with the review of the methods used to determine the simplification range of the influence structure. The authors' proposal of a modification of the DEMATEL method procedure and an example of its application are outlined in section 3. The paper ends with a summary and findings.

2. Determining the structure simplification range in the DEMATEL method

DEMATEL allows the structure of interactions unfolding between objects to be specified while taking into account their total, and that means both the direct and indirect, context. To this end, it uses information on direct interactions (Dytczak, Ginda 2015). The interactions between objects are considered in terms of influence. Therefore the structure of direct interactions is referred to as a direct-influence structure, while the structure representing the outcome of the method application as a total-influence structure. The range of simplification of the total influence structure is expressed by a parameter – threshold level of total influence θ. Its application enables one to eliminate the weakest interactions from the structure. Figure 1 shows a typical version of the DEMATEL calculation procedure taking into account the simplification of the total-influence structure through selected level θ.

Figure 1. A contemporary form of the DEMATEL method procedure



Start/accepting direct-influence structure/determining total-influence structure/specifying the simplification range of total-influence structure θ/simplification of total-influence structure/stop

Source: self-reported data.

Parameter θ is determined using two different approaches. The first one involves a discrete choice of the parameter. The impact of discretion is limited in the second approach by using information on total-influence structure. In order to estimate parameter θ, what is usually employed is the expected level of total influence and the number of standard deviations chosen subjectively. In implementing the second approach, the MMDE procedure proposed by Li and Tzeng (Li, Tzeng, 2009) compares favorably to others. It identifies an appropriate level of parameter θ drawing on the estimation of losses of information expressed by the total-influence structure which occur in the course of its simplification.

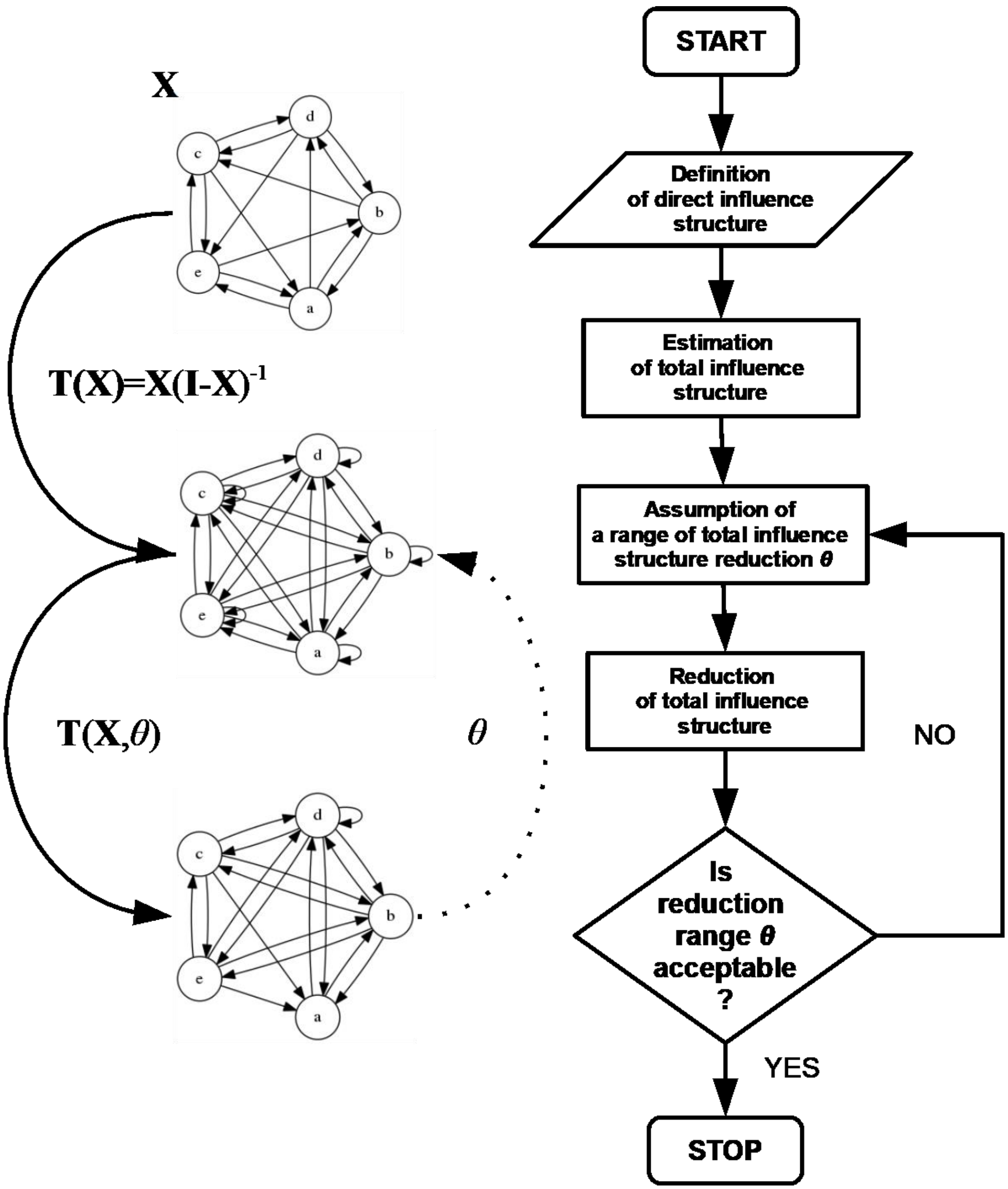
Using information on the total-influence structure does not foster the reliable evaluation of the effects of its simplification. That is because this information is secondary, since we obtain it using the DEMATEL method. Thus, the only reliable basis for evaluating the consequences arising from having the structure simplified is the original information expressed by direct-influence structure. It was therefore used while modifying the basic version of the calculation procedure of the DEMATEL method.

3. Modification of the DEMATEL method procedure

3.1 The modified procedure

Figure 2 presents a modified form of the procedure from Figure 1. This form allows for taking into account the consequences arising from the range accepted in the structure simplification. The modification consists in the initial estimation of the level of parameter θ which may then be modified gradually until the desired outcome. Details are discussed further on in the paper.

Figure 2. The proposal modifying the DEMATEL method procedure



Start/accepting direct-influence structure/determining total-influence structure/specifying the simplification range of total- influence structure θ/simplification of total- influence structure/acceptable simplification level/stop
Source: self-reported data.

3.2 Evaluation of the consequences of the structure simplification

In the evaluation of the consequences arising from the simplification of the total-influence structure we will transform the basic DEMATEL method formula:

$$\mathbf{T} = \mathbf{X}(\mathbf{I} - \mathbf{X})^{-1}, \tag{1}$$

in which: $\mathbf{T}$ denotes a *total-influence matrix* which expresses total-influence structure, $\mathbf{X}$ is a *standardized direct-influence matrix* expressing direct-influence structure, while $\mathbf{I}$ is a unit matrix.

$\mathbf{T}$, $\mathbf{X}$, $\mathbf{I}$ are square matrixes with the number of rows and columns equal to the number of interacting objects n.

The outcome of the transformation of formula (1) looks as follows:

$$\mathbf{X} = \mathbf{T}(\mathbf{I} - \mathbf{X}) \tag{2}$$

and it allows for the direct-influence structure to be expressed in terms of the total-influence structure. If the total-influence structure, simplified within the range determined by parameter θ, is denoted by symbol $\overline{\mathbf{T}}(\theta)$, then, by analogy to formula (2), we can determine its corresponding starting structure $\overline{\mathbf{X}}(\theta)$ in the following way:

$$\overline{\mathbf{X}}(\theta) = \overline{\mathbf{T}}(\theta)\left[\mathbf{I} - \overline{\mathbf{X}}(\theta)\right]. \tag{3}$$

The range within which the total-influence structure has been simplified should eliminate the differences between the structures expressed by matrixes $\overline{\mathbf{X}}(\theta)$ and **X.** Hence:

$$\overline{\mathbf{X}}(\theta) \approx \mathbf{X}. \tag{4}$$

Thanks to the application of (4) it is possible to eliminate matrix $\overline{\mathbf{X}}(\theta)$ from the right-hand side of formula (3). Thus, it is approximated by the following formula:

$$\overline{\mathbf{X}}(\theta) \approx \overline{\mathbf{T}}(\theta)\left[\mathbf{I} - \mathbf{X}\right] \tag{5}$$

3.3 Acceptance criterion of the structure simplification

The question remains open as to the form of the similarity criterion between the influence structures described using matrixes $\overline{\mathbf{X}}(\theta)$ and $\mathbf{X}$. In the first place, the application of a quantitative criterion seems a natural choice. With a view to estimating the differences between both structures, we can, for example, use Minkowski metric:

$$L\left[q,\overline{\mathbf{X}}(\theta),\mathbf{X}\right]=\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\left|\bar{x}_{ij}-x_{ij}\right|^{q}\right\}^{\frac{1}{q}}\leq E\ , \qquad (6)$$

where: q is a natural numerical parameter, while E denotes an acceptable positive real numerical max level of the difference between the structures.

The major advantage of the quantitative criterion (6) is its clarity. It is the parametric form of metric $L\left[q,\overline{\mathbf{X}}(\theta),\mathbf{X}\right]$ that makes it possible to obtain differentiated detailed versions, e.g. Manhattan distance metric (q=1), Euclidean distance metric (q=2) or specific Chebyshev distance ($q \to \infty$) which is equal to the maximum difference between the corresponding elements of matrix $\overline{\mathbf{X}}(\theta)$ and **X**:

$$L\left[\infty,\overline{\mathbf{X}}(\theta),\mathbf{X}\right]=\max_{i,j=1..n}\left|\bar{x}_{ij}-x_{ij}\right|. \qquad (7)$$

However, its implementation in practical terms gives rise to a number of considerable difficulties. First, a specific detailed form of metric must be chosen. Second, it is necessary to define the objective level of parameter E. Third, as a result of using the absolute value of difference $\left|\bar{x}_{ij}-x_{ij}\right|$ we lose the information on the differences in the directions of influence of objects. However, the most serious argument against the application of the quantitative criterion is the qualitative character of the DEMATEL method (Dytczak, Ginda 2013). That was the reason why eventually the qualitative criterion for the structure consistency was recommended.

For such evaluation of the similarity of structures the proposal was to use the criterion of the consistency of the influence directions in the structure approximated by formula (5) and in the direct-influence structure. To achieve this, we can use the graphic as well as matrix representation of the structures.

Since the elements of the direct-influence matrix are by definition nonnegative, while the application of formula (5) may lead to matrix $\overline{\mathbf{X}}(\theta)$having negative elements, before it can be used in the qualitative evaluation of the similarity of structures, it should be reduced to the form consistent with the direct-influence matrix, and so all nonnegative elements need to be eliminated from it. To this end let us note that negative influence of the j-th object on the i-th object ($\bar{x}_{ji}<0$) is identical with the influence of the i-th object on the j-th object ($\Delta\bar{x}_{ij}$) :

$$\underset{i,j=1..n}{\forall} \quad \Delta\bar{x}_{ij} = \begin{cases} \left|\bar{x}_{ji}\right| & \text{when} \quad \bar{x}_{ji} < 0\,, \\ 0 & when \quad \bar{x}_{ji} \geq 0\,. \end{cases} \tag{8}$$

Let us denote with symbol $\hat{\mathbf{X}}(\theta)$ the equivalent of matrix $\overline{\mathbf{X}}(\theta)$ whose form is consistent with the direct-influence matrix. Its elements are described by the following formula:

$$\underset{i,j=1..n}{\forall} \quad \hat{x}_{ij} = \begin{cases} \bar{x}_{ij} + \Delta\bar{x}_{ij} & \text{when} \quad \bar{x}_{ij} \geq 0\,, \\ \Delta\bar{x}_{ij} & \text{when} \quad \bar{x}_{ij} < 0\,. \end{cases} \tag{9}$$

Using the criterion of consistency of the direction of interactions while applying the matrixes describing the structures involves pair wise comparison of the sign of nonzero elements of the direct-influence matrix with their corresponding elements of matrix $\hat{\mathbf{X}}(\theta)$. The condition for the structure consistency can be formally given as follows (i, j = 1,2…n):

$$\underset{i,j=1,2..n}{\forall} \quad \underset{x_{ij}>0}{\forall} \quad sign(\hat{x}_{ij}) = sign(x_{ij}), \tag{10}$$

where *sign* denotes the function sign:

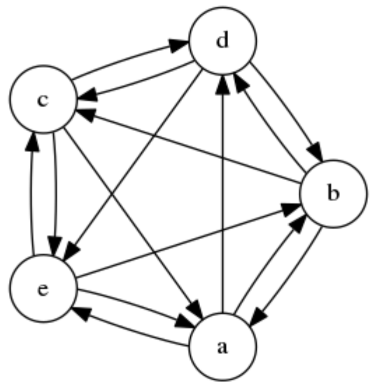
$$sign(y) = \begin{cases} +1 & \text{when} \quad y > 0, \\ 0 & \text{when} \quad y = 0, \\ -1 & \text{when} \quad y < 0. \end{cases} \tag{11}$$

The structure expressed by $\hat{\mathbf{X}}(\theta)$ also provides the basis for using the criterion of the influence direction consistency drawing on the graphic representations of the structures.

3.4 A calculation example

We will illustrate the application of the criterion of the qualitative evaluation of the similarity of the structures employing a calculation example. In the example, we examine the identification of the structure of interactions among certain five objects. They are denoted with small letters: a, b, c, d and e. The direct-influence structure is illustrated in Figure 3 and matrix $\mathbf{X}^*$ (12). Marked in bold are those elements in the matrix which correspond to the interactions adopted between objects a-e. They are precisely what provides the basis for the verification of the range within which the total-influence structure is simplified.

Figure 3. An example of direct-influence structure



Source: self-reported data.

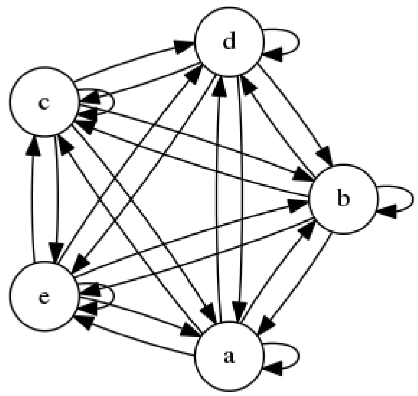
$$\mathbf{X}^* = \begin{bmatrix} 0 & \mathbf{1} & 0 & \mathbf{3} & \mathbf{1} \\ \mathbf{2} & 0 & \mathbf{3} & \mathbf{1} & 0 \\ \mathbf{1} & 0 & 0 & \mathbf{2} & \mathbf{3} \\ 0 & \mathbf{3} & \mathbf{2} & 0 & \mathbf{2} \\ \mathbf{3} & \mathbf{2} & \mathbf{1} & 0 & 0 \end{bmatrix}. \quad (12)$$

After dividing the matrix by the maximum sum of horizontal entries of its elements, which is 7, we obtain direct-influence matrix **X**. By substituting it into the formula (1) we obtain a total-influence structure expressed by matrix **T**.

$$\mathbf{T} = \begin{bmatrix} 0.915 & 1.118 & 0.992 & 1.264 & 1.060 \\ 1.249 & 1.017 & 1.371 & 1.215 & 1.113 \\ 1.215 & 1.118 & 1.042 & 1.264 & 1.410 \\ 1.269 & 1.531 & 1.488 & 1.188 & 1.444 \\ 1.351 & 1.215 & 1.108 & 1.069 & 0.974 \end{bmatrix}. \quad (13)$$

Its graphic illustration is presented in Figure 4.

Figure 4. The total-influence structure obtained



Source: self-reported data.

Initially let us use the simplification method for total-influence structure that is proposed in the work (Ginda 2015). It consists in retaining, in the simplified structure, the strongest total interactions making up altogether 80% of the share in all the interactions between objects. In order to identify the strongest influence, Table 1 compares all the interactions in descending order. It turns out that the 80%-share has 19 strongest interactions, with its corresponding level of parameter θ being at 1.069.

Table 1. Comparison of interactions

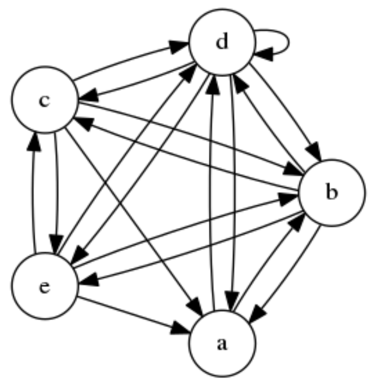
No	Interaction	Element **T**	Strength	Cumulative sum	Cumulative share
1	d – b	t_{42}	1.531	1.531	5.1%
2	d - c	t_{43}	1.487	3.018	10.1%
3	d – e	t_{45}	1.444	4.462	14.9%
4	c – e	t_{35}	1.410	5.872	19.6%
5	b – c	t_{23}	1.371	7.243	24.1%
6	e – a	t_{51}	1.351	8.594	28.6%
7	d – a	t_{41}	1.269	9.863	32.9%
8	c – d	t_{34}	1.264	11.127	37.1%
9	a – d	t_{14}	1.264	12.391	41.3%
10	b – a	t_{21}	1.249	13.640	45.5%
11	e – b	t_{52}	1.215	14.856	49.5%
12	c – a	t_{31}	1.215	16.071	53.6%
13	b – d	t_{24}	1.215	17.286	57.6%
14	d – d	t_{44}	1.187	18.473	61.6%
15	c – b	t_{32}	1.118	19.591	65.3%
16	a – b	t_{12}	1.118	20.709	69.1%
17	b – e	t_{25}	1.113	21.822	72.7%
18	e – c	t_{53}	1.108	22.930	76.4%
19	e – d	t_{54}	1.069	24.000	80.0%
20	a – e	t_{15}	1.060	25.060	83.5%
21	c – c	t_{33}	1.042	26.102	87.0%

22	b – b	t_{22}	1.017	27.119	90.4%
23	a – c	t_{13}	0.992	28.111	93.7%
24	e – e	t_{55}	0.974	29.085	97.0%
25	a – a	t_{11}	0.915	30.000	100.0%

Source: self-reported data.

Thus, Figure 5 shows the identified and initially simplified total-influence structure which was obtained after having eliminated interactions below the threshold level of total influence at 1.069. It is expressed by the following matrix.

Figure 5. The initially simplified total-influence structure $\overline{\mathbf{T}}(\theta = 1.069)$



Source: self-reported data.

$$\overline{\mathbf{T}}(\theta = 1.069) = \begin{bmatrix} 0 & 1.118 & 0 & 1.264 & 0 \\ 1.249 & 0 & 1.371 & 1.215 & 1.113 \\ 1.215 & 1.118 & 0 & 1.264 & 1.410 \\ 1.269 & 1.531 & 1.488 & 1.188 & 1.444 \\ 1.351 & 1.215 & 1.108 & 1.069 & 0 \end{bmatrix}. \tag{14}$$

Hence, on the basis of formula (5) we obtain the approximation of matrix $\overline{\mathbf{X}}(\theta)$:

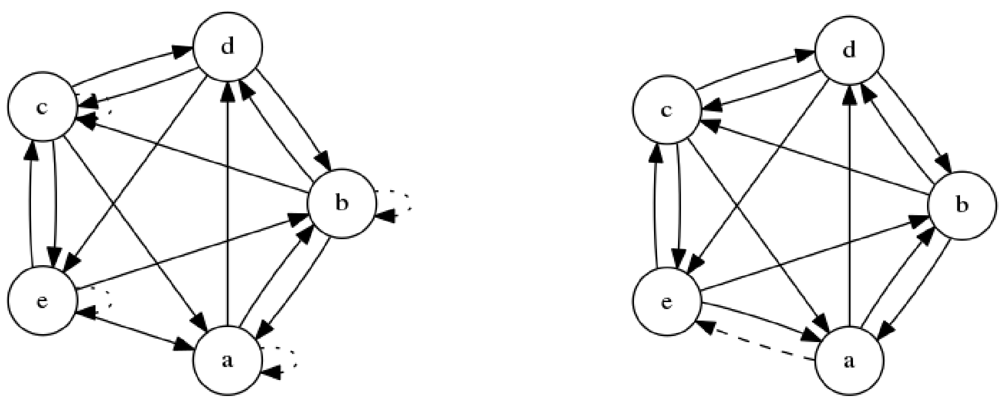
$$\overline{\mathbf{X}}(\theta = 1.069) = \begin{bmatrix} -0.391 & +0.576 & -0.840 & +1.104 & -0.361 \\ +0.576 & -1.017 & +0.865 & +0.288 & 0 \\ +0.292 & 0 & -1.042 & +0.538 & +0.875 \\ 0 & +0.429 & +0.286 & 0 & +0.286 \\ +1.846 & +0.564 & +0.282 & 0 & -0.974 \end{bmatrix}. \tag{15}$$

Having applied formula (9) to eliminate its negative elements, we obtain the following form of matrix $\hat{\mathbf{X}}(\theta)$:

$$\hat{\mathbf{X}}(\theta = 1{,}069) = \begin{bmatrix} +0.391 & +\mathbf{0.576} & 0 & +\mathbf{1.104} & \mathbf{0} \\ +\mathbf{0.576} & +1.017 & +\mathbf{0.865} & +\mathbf{0.288} & 0 \\ +\mathbf{1.132} & 0 & +1.042 & +\mathbf{0.538} & +\mathbf{0.875} \\ 0 & +\mathbf{0.429} & +\mathbf{0.286} & 0 & +\mathbf{0.286} \\ +\mathbf{2.207} & +\mathbf{0.564} & +\mathbf{0.282} & 0 & +0.974 \end{bmatrix}. \qquad (16)$$

Marked in bold are the elements of the matrix that are compared with the matrix representation of the total-influence. Figure 6 illustrates the differences between the graphic representations of the structure expressed by matrix (16) and the direct-influence structure from Figure 3. The continuous line shape expresses interactions present in the direct-influence structure.

Figure 6. The illustration of the consequences of the initial simplification of total-influence structure



a) structure expressed by matrix $\hat{\mathbf{X}}(\theta = 1.069)$ b) direct-influence structure

Source: self-reported data.

The comparison of the elements in bold of matrixes $\hat{\mathbf{X}}(\theta = 1.069)$ and $\mathbf{X}^*$ (12) shows that they differ in the sign of one element. This element expresses the influence of object “a” on object “e”. The same conclusion can be drawn on the basis of the comparison between Figures 6a) and 6b). In the simplified structure the lack of the influence initially assumed of object “a” on object “e” is very clear, which in Figure 6b) is symbolized by a curve drawn in a dashed line. The findings of the above comparisons demonstrate that the threshold level θ = 1.069 brings about a too simplified total-influence structure.

According to the diagram depicted in Figure 2, the simplification range with respect to the total-influence structure should now be limited by reducing level θ. This can be achieved by gradually enhancing the simplified structure with the interactions next in the ranking presented in Table 1. In the case in question we would enhance the simplified structure by interactions ranked twentieth, and if necessary, also those ranked next.

Eventually it turned out that we would obtain a proper, not excessively simplified, total-influence structure by adding to the structure presented in Figure 5 the influence of object "a" on "e". Level $\theta = 1.06$ corresponds to this situation.

4. Summary and findings

The original proposal presented herein and referring to the enhancement of the calculation procedure by providing support in defining the simplification range of complex total-influence structures eliminates a serious drawback of the DEMATEL method. Its major advantage is its reliability in terms of the evaluation of consequences arising from the simplification of structures which is achieved by referring them to the basic information on direct influence exerted by objects. Further advantages arise from the flexible nature of the method enhancement. It is because it imposes neither specific form of the criterion for acceptance of the simplified total-influence structure nor any methods of estimation, modification and the way in which the structure simplification range is approximated.

Considering the importance surrounding the issue of reliable estimation of the simplification range of structures, in the future the authors intend to extend the feasibility analysis of the proposal demonstrated for complex issues. Moreover, it is worth applying it in testing the usefulness of available methods employed to define the range within which structures are simplified and the criteria for the evaluation of acceptance and approximation of appropriate simplification range.

The authors also hope that this paper will contribute to and launch meaningful discussion on the methods used to test acceptance and to specify the simplification range of structures.

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